Distributed algorithms and local structures in graphs

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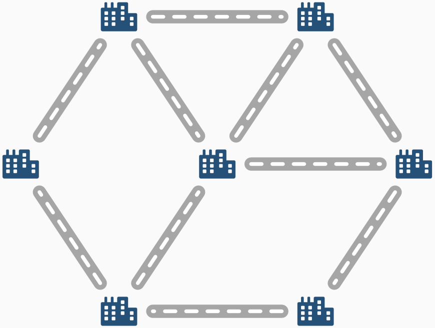
23 June 2026

Under supervision of Marthe Bonamy and Cyril Gavoille

Building hospitals

Goal: every city should have a hospital or one next to it.



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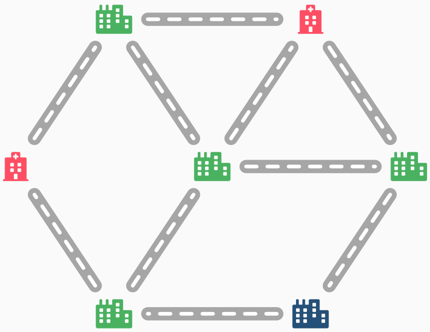

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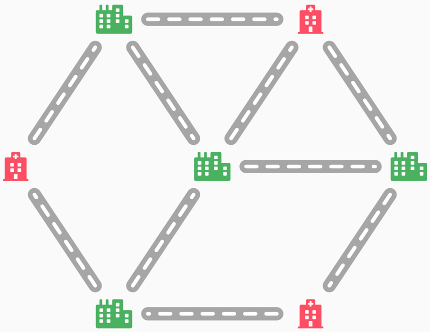

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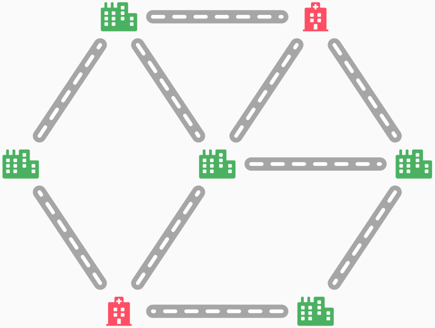
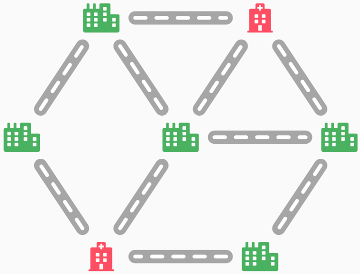


Minimize the number of hospitals.

Minimum Dominating Set



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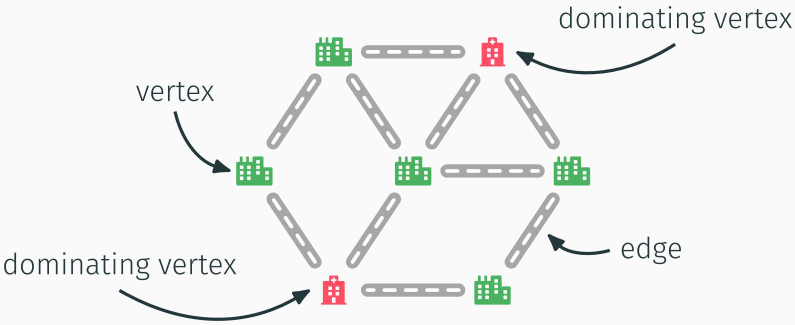

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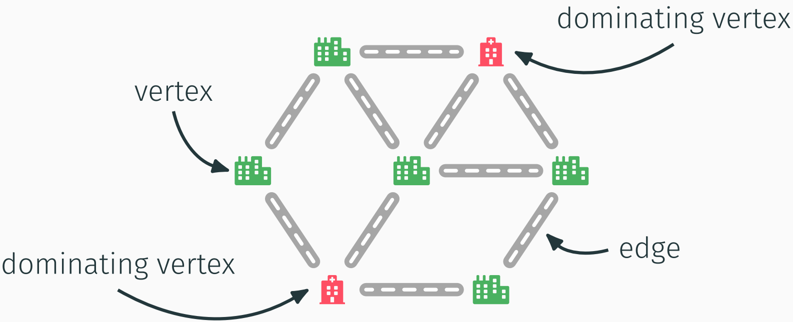

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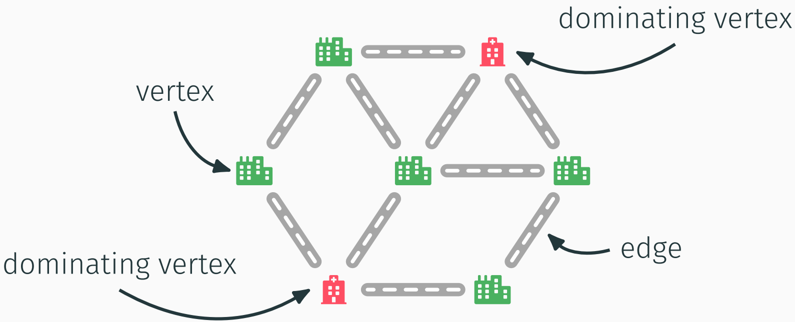
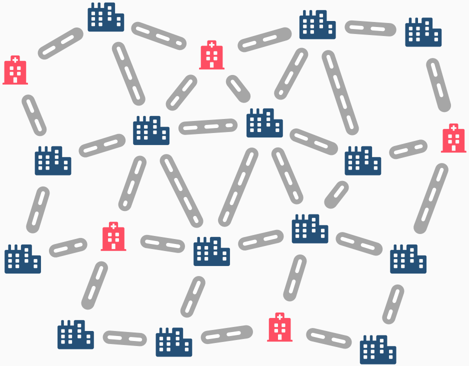


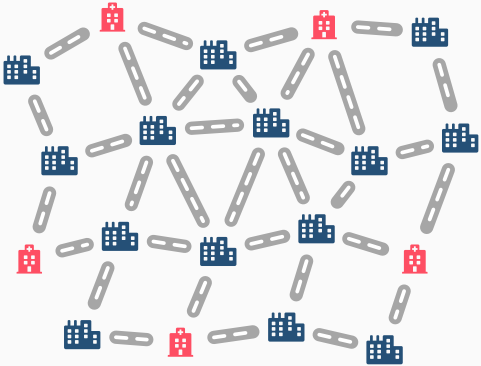
- Every vertex is a dominating vertex or neighbors one
- Choose as few dominating vertices as possible

Hardness with global knowledge



Central planner

Hardness with global knowledge



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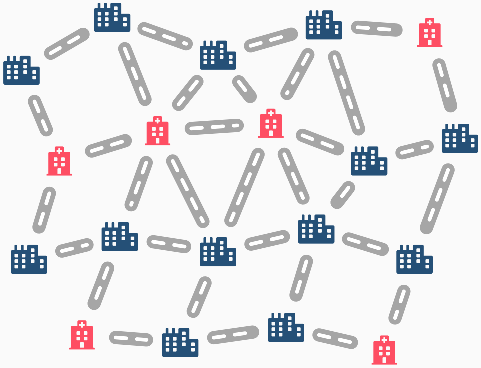

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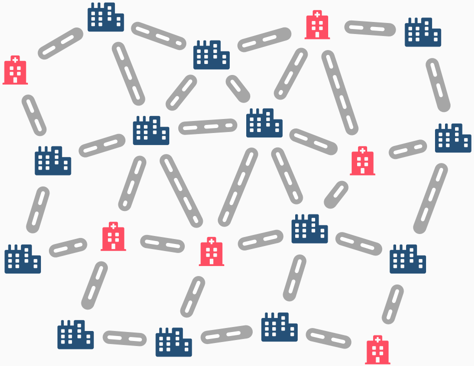

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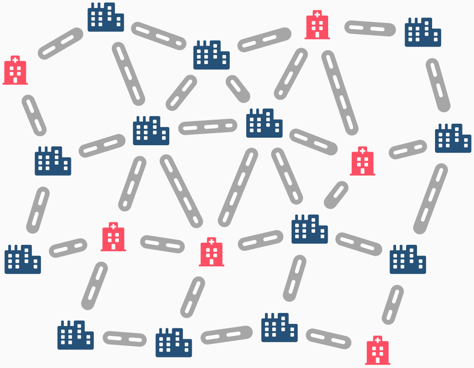


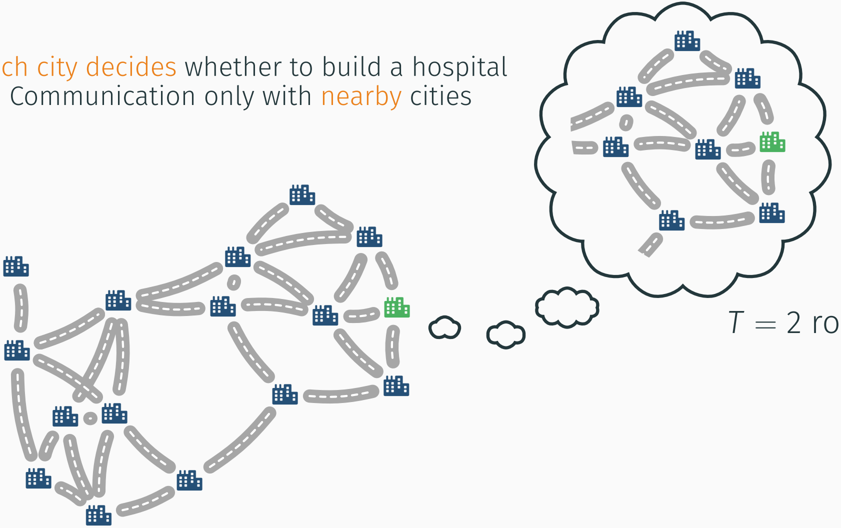
Central planner

- computing an optimal solution is difficult (NP-hard)
- greedy algorithm: $(2 + \log \Delta)$-approximation, essentially tight

The more realistic LOCAL model

[Linial 1992]

Each city decides whether to build a hospital
Communication only with nearby cities



$T = 2$ rounds

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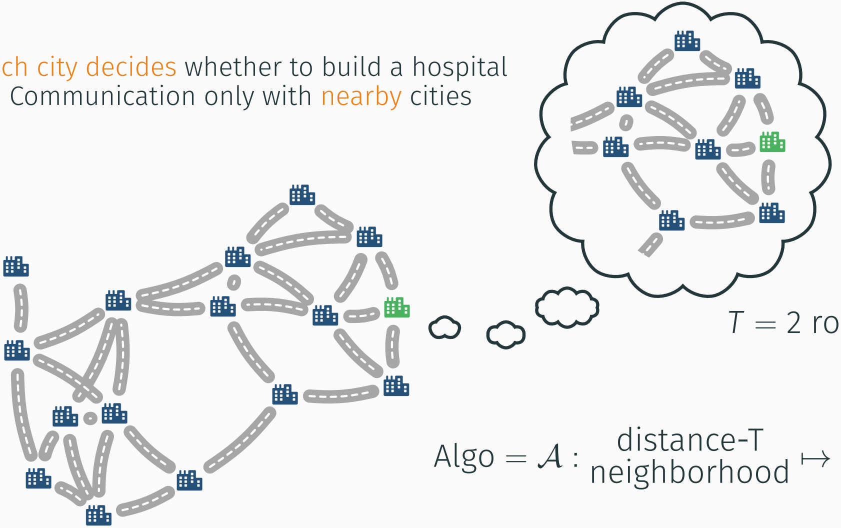

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$$\text{Algo} = \mathcal{A} : \begin{array}{c}\text{distance-T}\\ \text{neighborhood}\end{array} \mapsto \begin{array}{c}\text{membership to}\\ \text{returned DS}\end{array}$$

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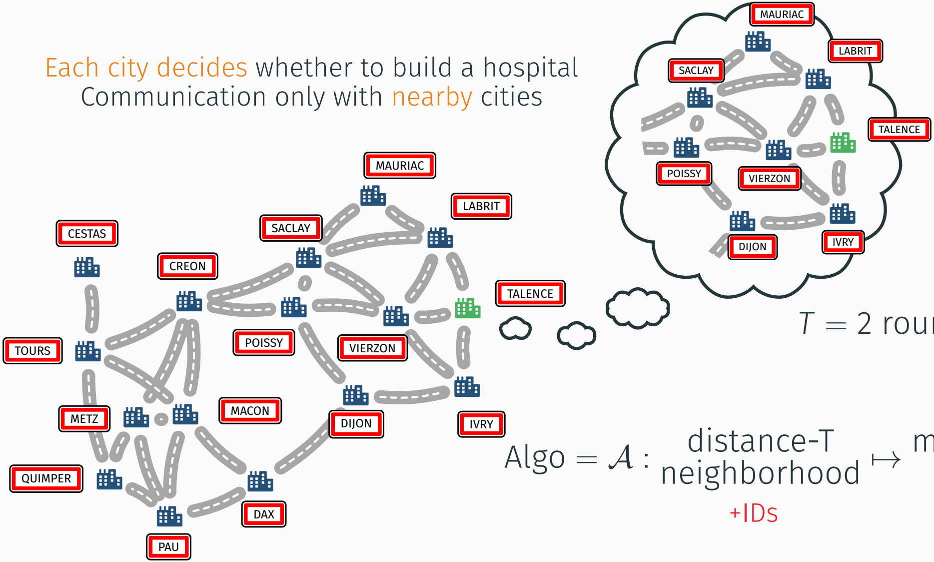

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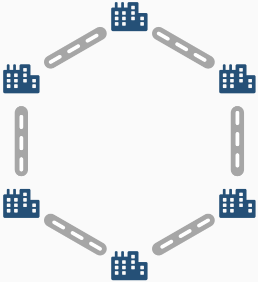
Algo $= \mathcal{A}$: distance-T neighborhood +IDs $\mapsto$ membership to returned DS

A naive LOCAL algorithms

Naive algo: construct if no nearby hospital



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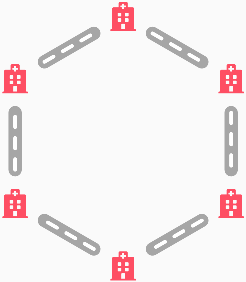

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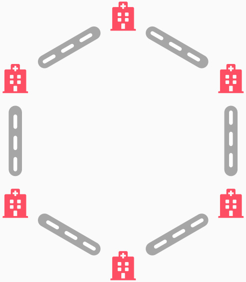
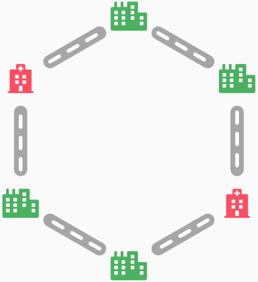


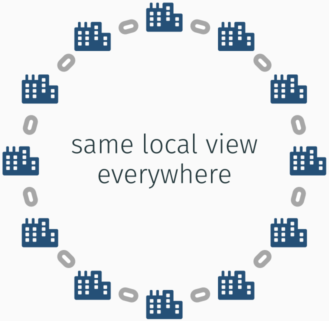


Result

Far from optimal on a cycle.

Why cycles are hard in constant time

The distance-r neighborhoods are all paths of length $2r + 1$



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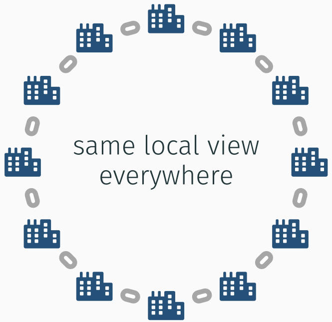

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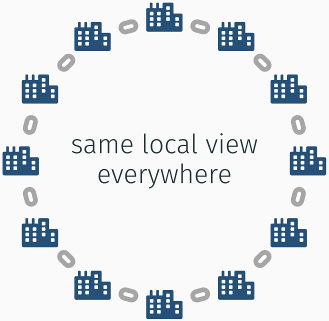

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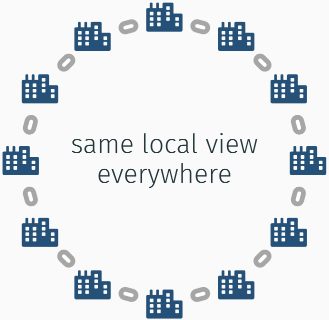

Intuition

Local symmetry prevents $(3-\varepsilon)$-approximations on cycles.

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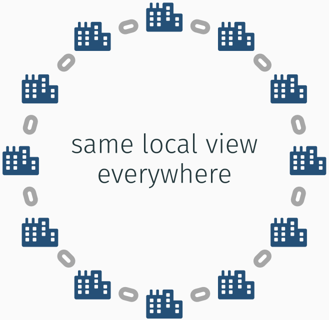

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Can be made formal with identifiers HIESSE !

Is all hope lost?

Theorem (Kuhn, Moscibroda, and Wattenhofer – 2004)

*It is **impossible** to approximate Minimum Dominating Set with a constant number of rounds and constant approximation ratio on **general graphs**.*

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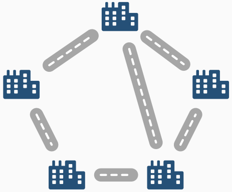
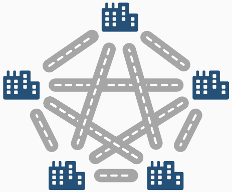
💡 **Restricting the graph class!** 💡

Planar graphs

Planar



Non planar



Planar: can be drawn on the plane without crossing edges

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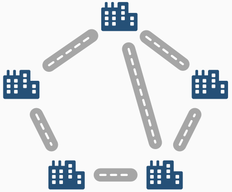

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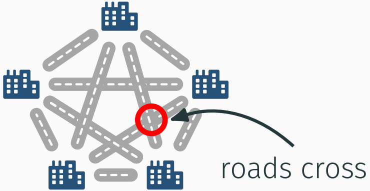

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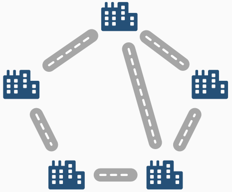

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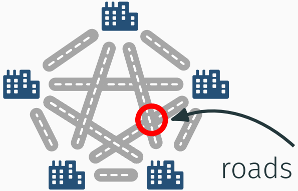

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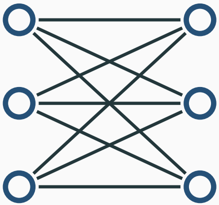
Theorem (Heydt, Kublenz, Ossona de Mendez, Siebertz, Vigny – 2025)

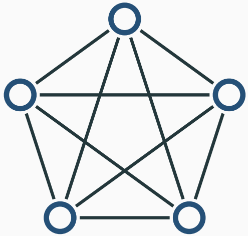
There exists a $(11+\varepsilon)$-approximation for MDS on planar graphs, running in $f(\varepsilon)$ rounds.

Wagner’s theorem

$K_{3,3}$



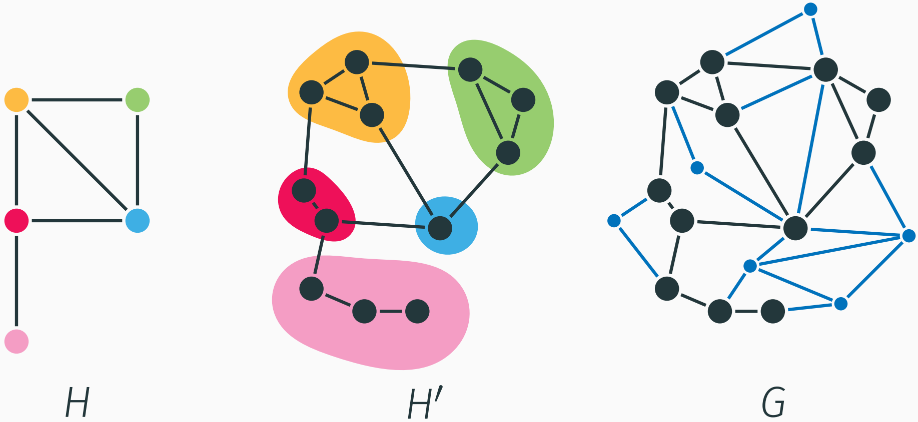
K_5



Theorem (Wagner – 1937)

A graph is planar if and only if it contains no K_5 and $K_{3,3}$ as minors.

Graph minors



H is a minor of G

State of the art for MDS with constant LOCAL rounds

MINOR-FREE CLASSES	APPROX. RATIO		#rounds
	lower	upper	
trees (K_3)	3	3	1
outerplanar (K_4, $K_{2,3}$)	5	5	2
planar (K_5, $K_{3,3}$)	7	$11+\varepsilon$	$f(\varepsilon)$

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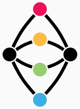

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$K_{2,t}$-minor-free	5	$\mathbf{2t - 1}$	**3**
	5	$\Theta(t) \to$ **50**	$\boldsymbol{f(t)}$
$K_{3,t}$-minor-free	7	$(2 + \varepsilon) \cdot (t + 4)$	$f(\varepsilon, t)$
$K_{s,t}$-minor-free	7	$t^{\mathcal{O}(st\sqrt{\log s})}$	$f(t)$

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[1] **PODC' 2026**: M. Bonamy, A. Das, C. Gavoille, TP, J. Suomela, A. Wesolek. *Meta-Theorems for Cuttable Distributed Problems*

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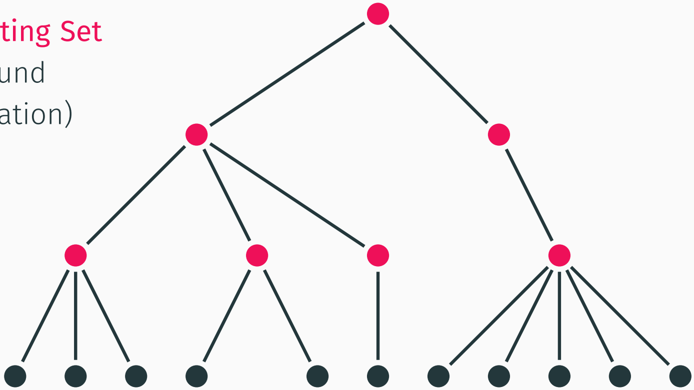
Example 1: trees

- Dominating Set (one-round computation)



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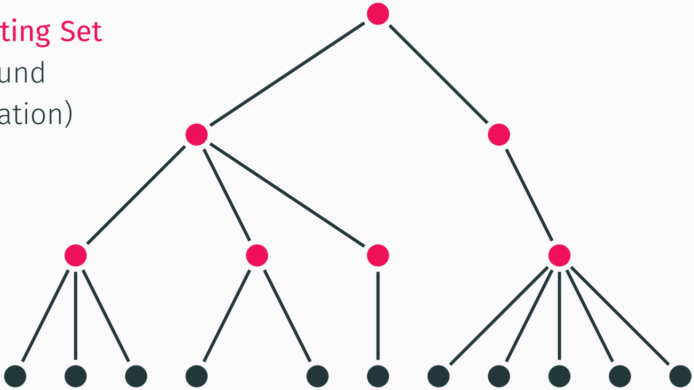

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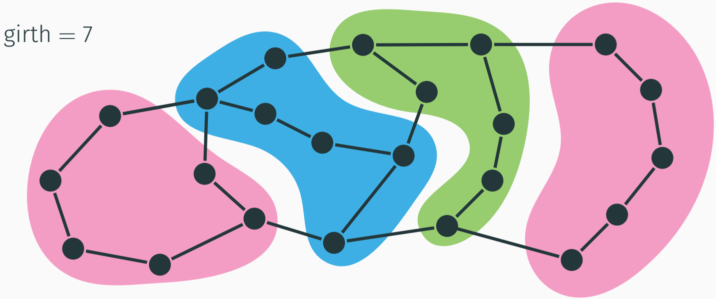
Theorem (folklore)

$|\{v \in V(T) \mid \deg(v) \geq 2\}| \leq 3 \cdot \mathrm{MDS}(T)$

Example 2: high girth graphs



Reusing the algorithm of trees?

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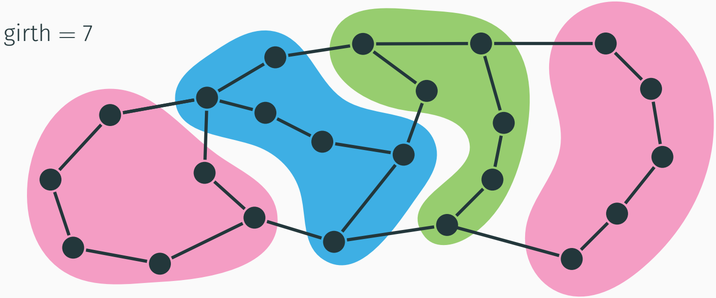

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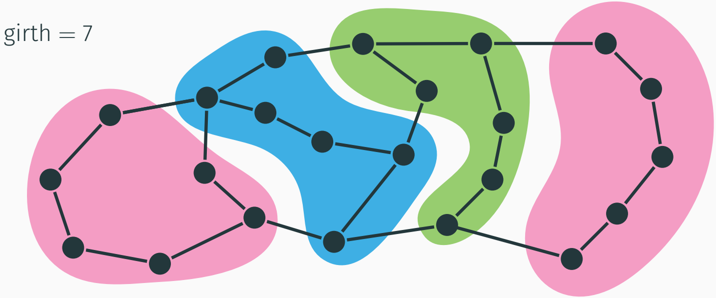

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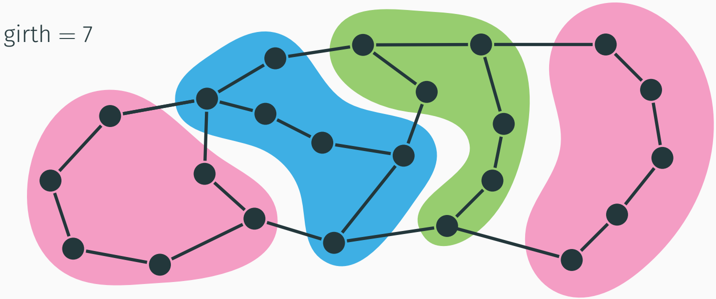

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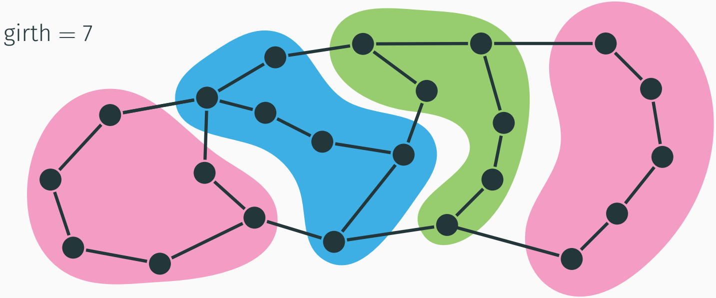

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- Diameter $\leq$ girth/2 ($\implies$ to see a tree)
- Spacing bags of same color ($\implies$ no overcounting for fixed color)
- Few colors ($\implies$ to limit overcounting)

Asymptotic dimension [Gromov 1993]

Asymptotic dimension of $\mathcal{C}$ is d if $\exists f : \mathbb{N} \to \mathbb{N}, \forall G \in \mathcal{C}, \forall r \in \mathbb{N}, \exists B_1, B_2, \dots \subseteq V(G)$, such that

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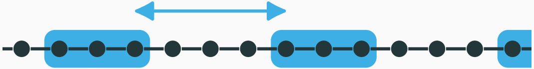
- **Disjointness:** if $c(B_i) = c(B_j)$, then $\mathsf{dist}(B_i, B_j) > r$



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- **Cover:** $V(G)$ is partitionned by the B_i's

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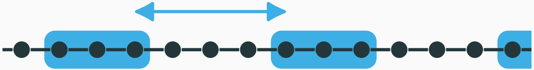
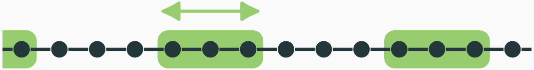
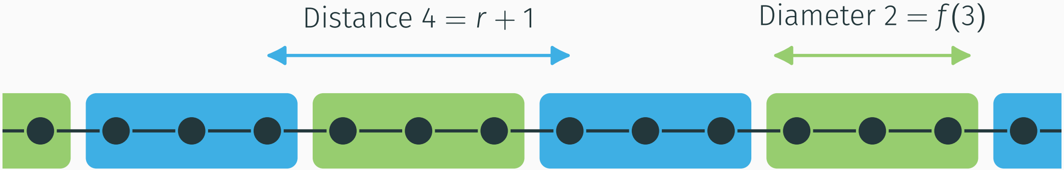
- **Disjointness:** if $c(B_i) = c(B_j)$, then $\mathbf{dist}(B_i, B_j) > r$



- **Boundedness:** $\forall i, \mathbf{diam}_G(B_i) \leq f(r)$



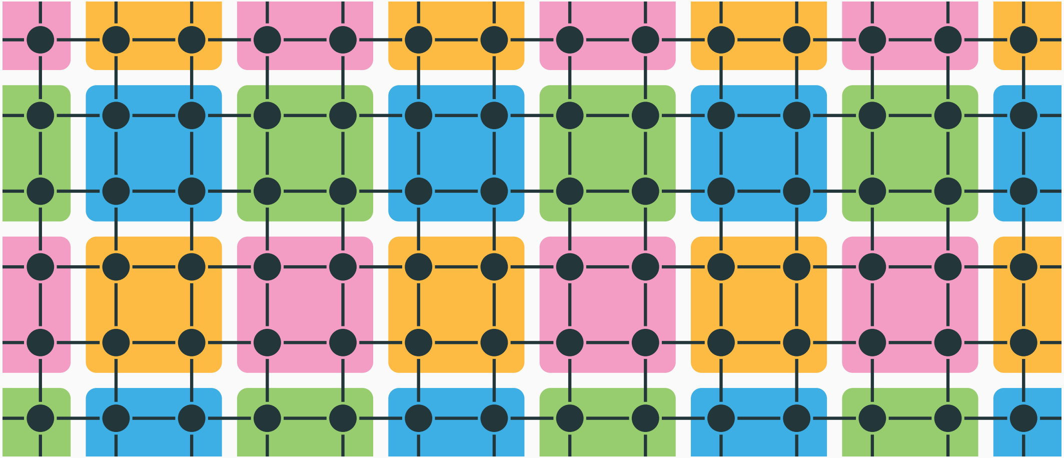
Example 1: the path ($r = 3$)



Dimension $= 1$ (2 colors)

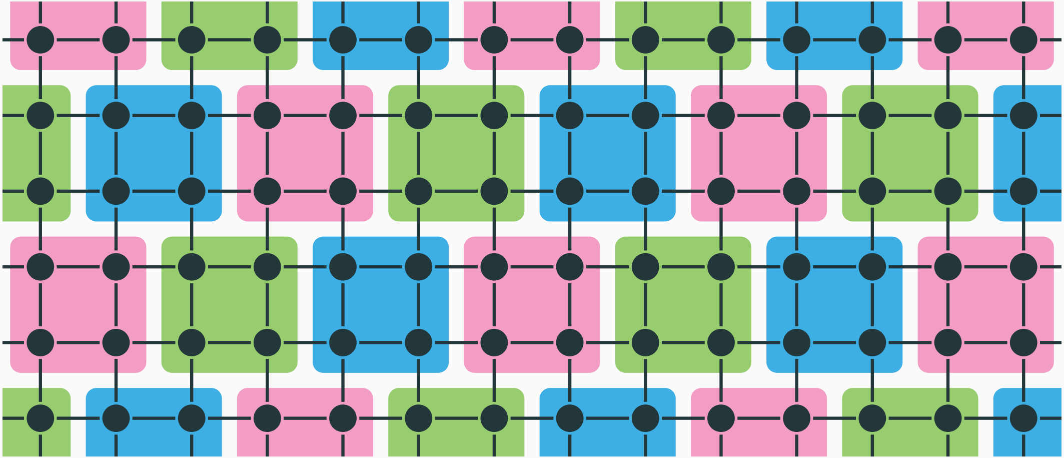
with $f(r) = r - 2$

Example 2: the grid – attempt 1 ($r = 2$)



Dimension ≤ 3

Example 2: the grid – attempt 2 ($r = 2$)



Dimension $= 2$!

Theorem (Bonamy, Bousquet, Esperet, Groenland, Liu, Pirot, Scott – 2020)

Every class excluding a fixed minor has asymptotic dimension ≤ 2.

Application: distributed algorithms

How to use graph theory in distributed algorithms?

How to use graph theory in distributed algorithms?

Global concept

$\downarrow$

Local concept

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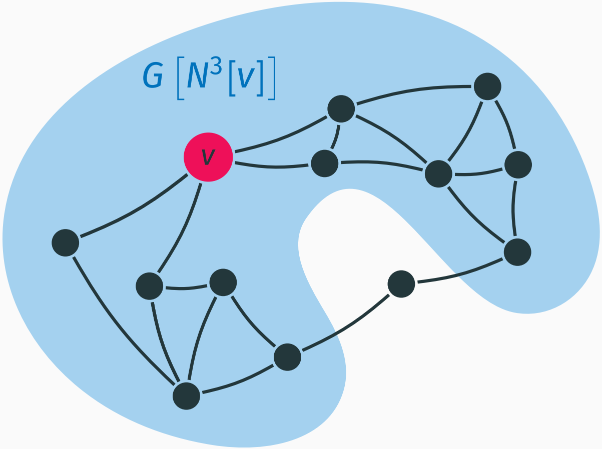
Definition

v is a *r-local cutvertex* if v is a cutvertex of $G\left[N^r[v]\right]$.



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Theorem (Bonamy, Gavoille, P, Wesolek, PODC'25)

On $K_{2,t}$-minor-free graphs, there is a 50-approximation of MINIMUM DOMINATING SET *in the LOCAL model, running in $f(t)$ rounds.*

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Asymptotic dimension only used in the analysis!

The algorithm

$$Y_t(G) = \bigcup\{18t\text{-local cuts of size } \leq 2\} \setminus \{\text{non-interesting vertices}\}$$

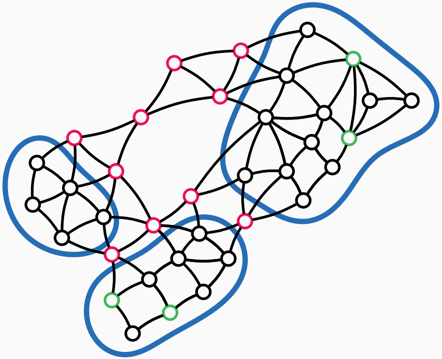
$$\text{Algorithm} = Y_t(G) \cup [\text{brute-force on } G - Y_t(G)]$$

Lemma 1

$|Y_t(G)| = \mathcal{O}(\text{asdim}) \cdot \text{MDS}(G)$.

Lemma 2

If G is $K_{2,t}$-minor-free, every connected component of $G - Y_t(G)$ has diameter $g(t)$.



Another application: meta-theorems

Theorem (Bonamy, Das, Gavoille, P, Suomela, Wesolek, PODC'26)

On graphs of Euler genus g, there exists a $(34+\varepsilon)$-approximation of Minimum Dominating Set *in the LOCAL model, in $f(g,\varepsilon)$ rounds.*

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Meta-theorem (baby version)

Any constant-round α-approximation for MDS on planar graphs can be turned into a $f(g)$-round $(3\alpha+1)$-approximation algorithm for MDS on Euler genus-g graphs.

$$\boxed{\mathcal{A}} \xrightarrow{\text{meta-theorem}} \boxed{\mathcal{B}}$$

$\mathcal{A}$	$\mathcal{B}$
α-approx	$(3\alpha+1)$-approx
planar graphs	genus g
constant-round	$f(g)$-rounds

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$11+\varepsilon$
(Heydt et al.)

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$34+\varepsilon$

α-approx
planar graphs
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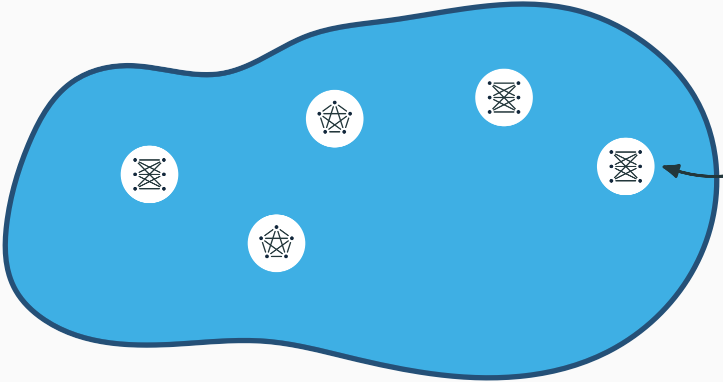
The algorithm $\mathcal{B}$

$T := f(2\rho + 2) + \rho + 2$



local
non-planarities

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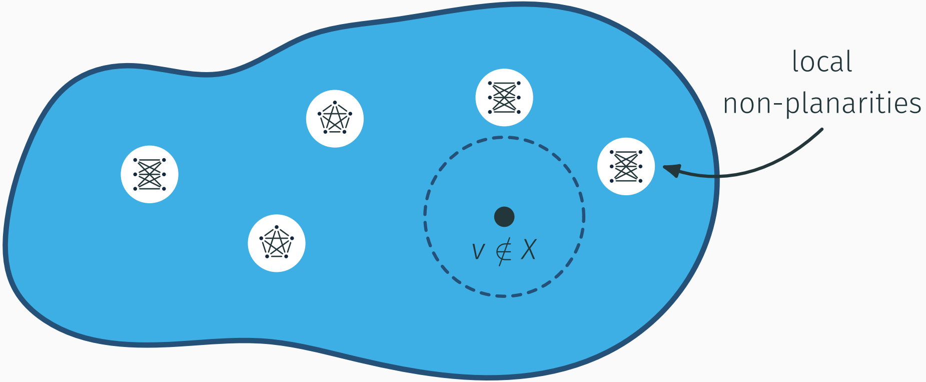

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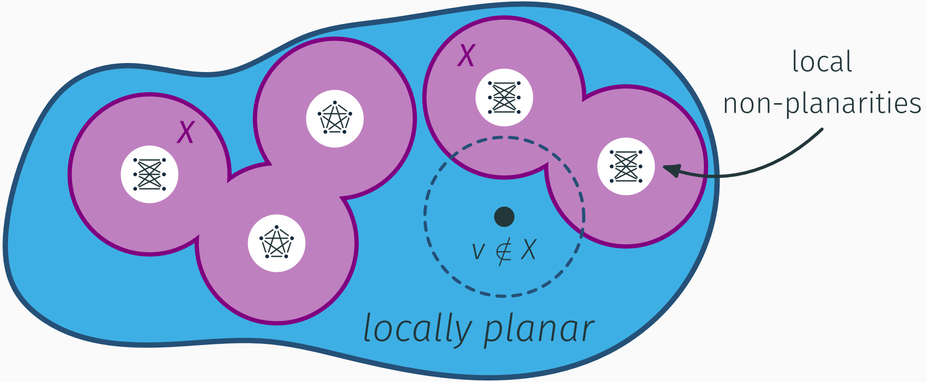

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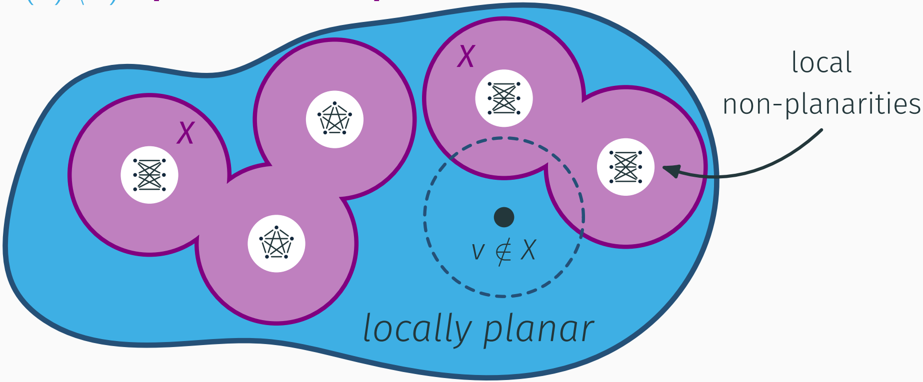

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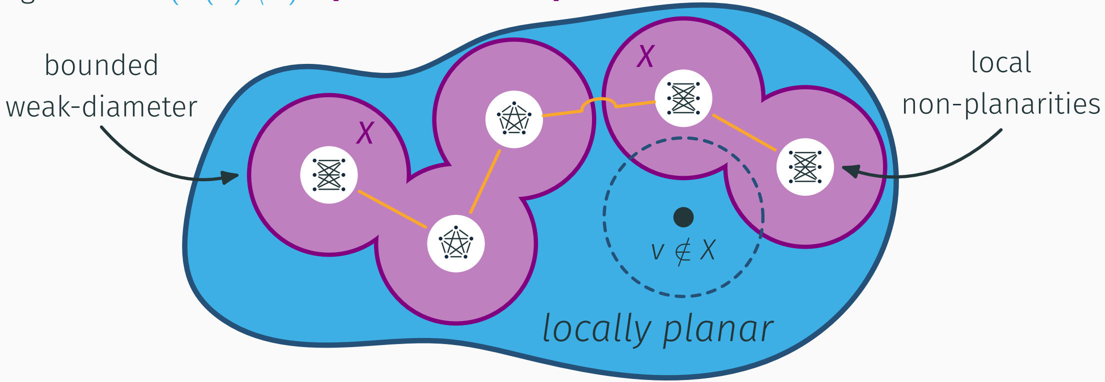

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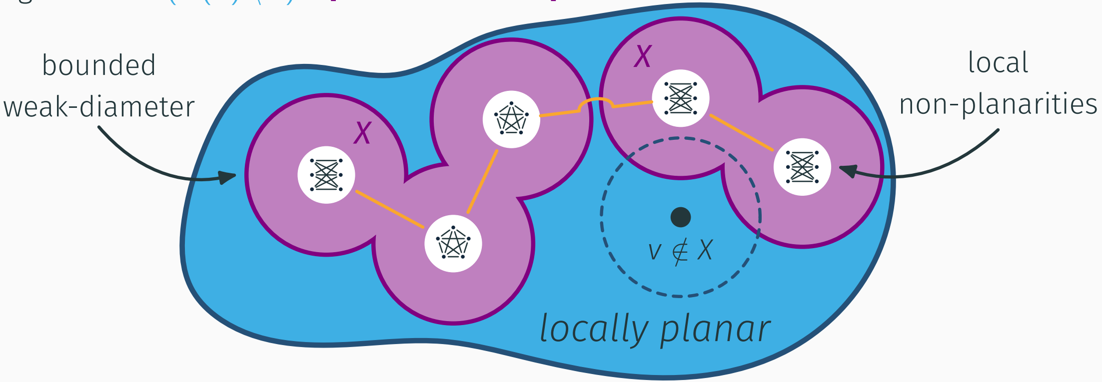

$\mathrm{genus}(K_{3,3} \cup K_5 \cup K_5 \cup \cdots \cup K_{3,3}) \geq \mathrm{genus}(K_{3,3}) + \mathrm{genus}(K_5) + \cdots + \mathrm{genus}(K_{3,3})$

Proof: outside the error set

$\mathcal{A}$ a ρ-round α-approx on planar graphs.

Goal: bound $|(\mathcal{A}(G) \cap \text{bag}) \setminus X|$.

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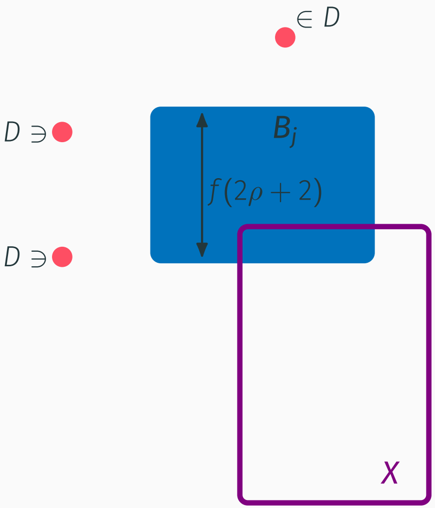

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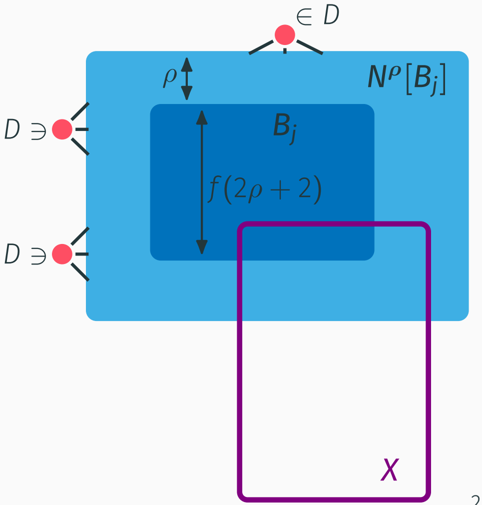

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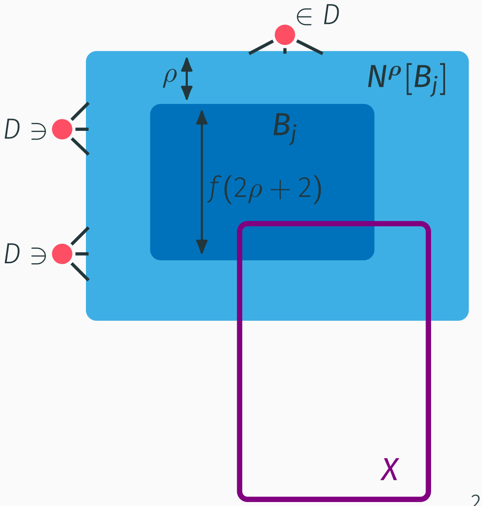

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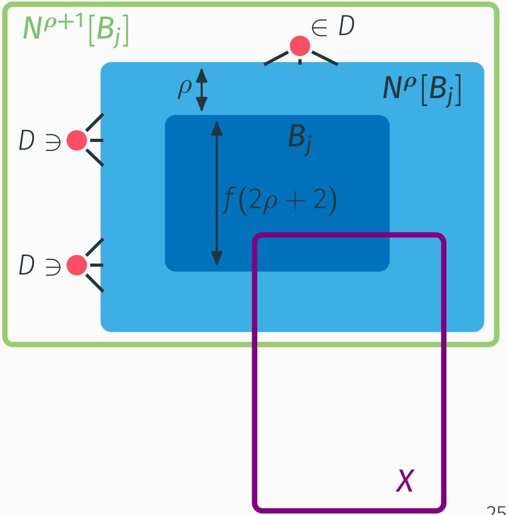

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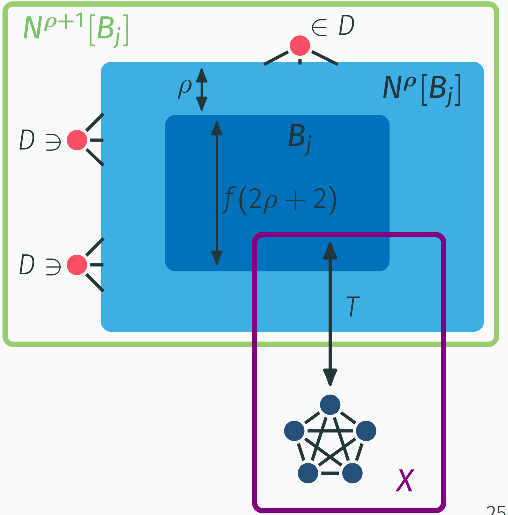

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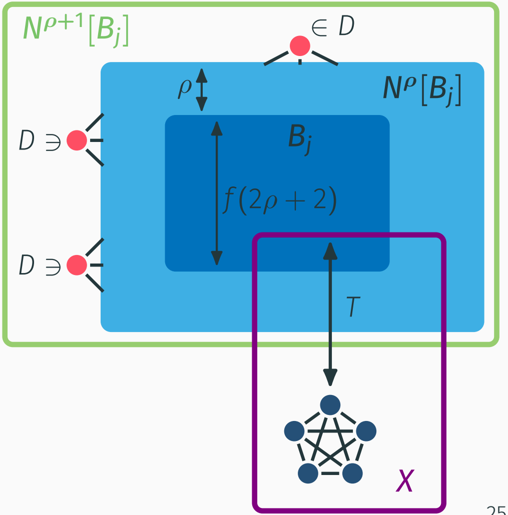
$$|(\mathcal{A}(G) \cap B_j) \setminus X| \overset{1}{=} |(\mathcal{A}(G') \cap B_j) \setminus X| \leq |\mathcal{A}(G')|$$
$$\overset{3}{\leq} \alpha \cdot \text{MDS}(G') \overset{2}{\leq} \alpha \cdot |D \cap N^{\rho+1}[B_j]|$$



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(the $N^{\rho+1}[B_j]$ are disjoint)

$$|\mathcal{A}(G) \setminus X| \leq \alpha \cdot \sum_{i=0,1,2} |D| = 3\alpha \cdot \mathrm{MDS}(G)$$

Conclusion and perspectives

Follow-up works:

- H-minor-free graphs admit a 50-approximation if $\mathsf{pathwidth}(H) = 2$.
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- On problems other that MDS.

? H-minor-free $\rightarrow$ LOCAL $\mathcal{O}(\mathsf{pathwidth}(H))$-approximation in constant time ? (would be optimal)

? What is the right approximation ratio for planar graphs?

? What about distance-k MDS?

Other works not included here

- distributed algorithms [2,4,9]
- structural graph theory [7]
- induced minors [6]
- extremal graph theory [5]
- property testing [8]
- FPT algorithms [3]
- temporal graphs [1]

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