Meta-Theorems for Cuttable Distributed Problems

PODC 2026, Egham

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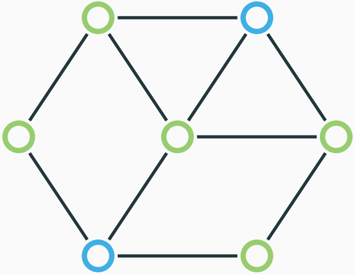
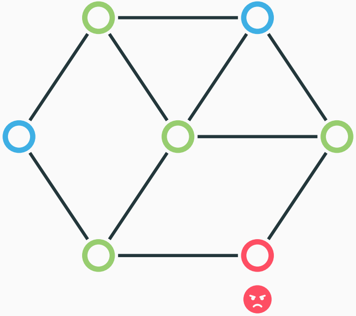
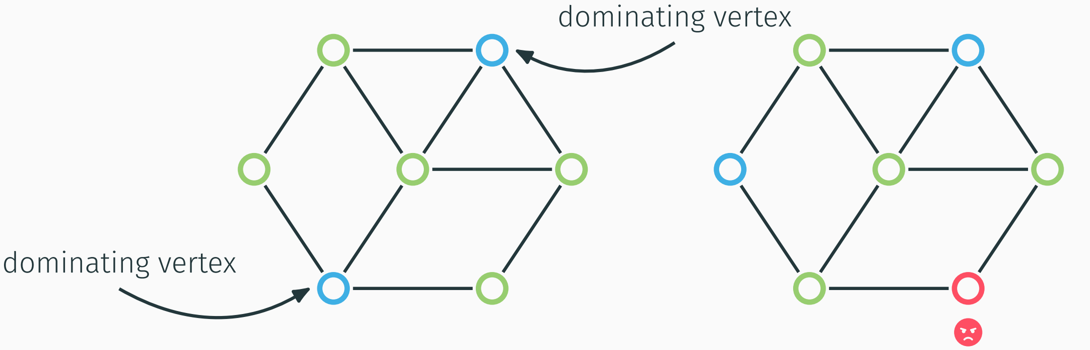
Minimum Dominating Set






dominating vertex
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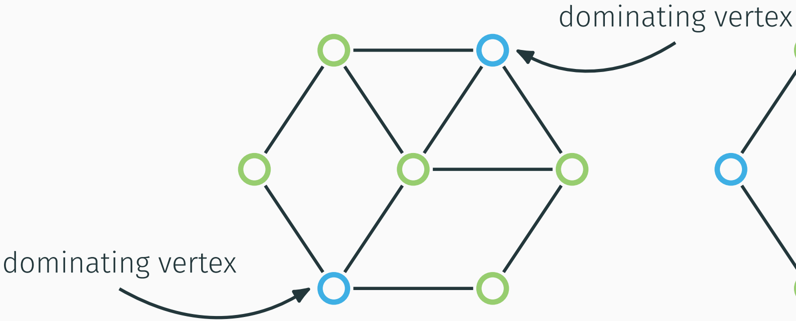

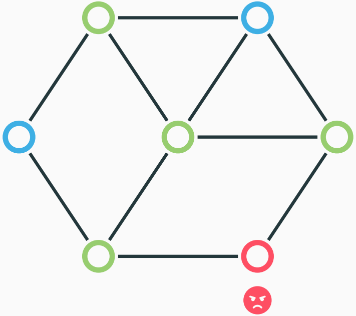

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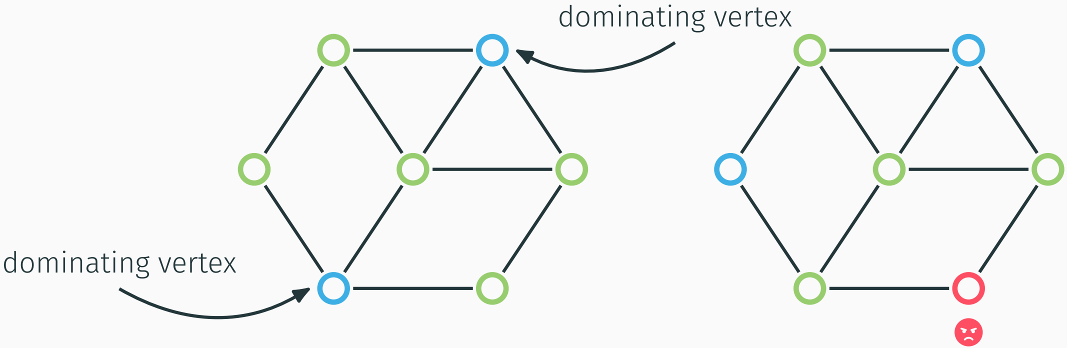
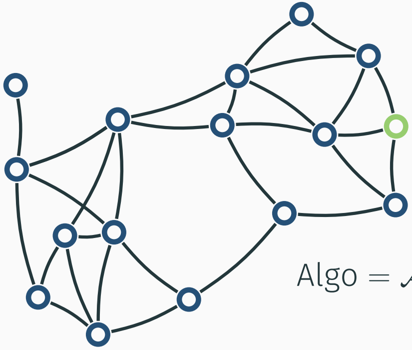


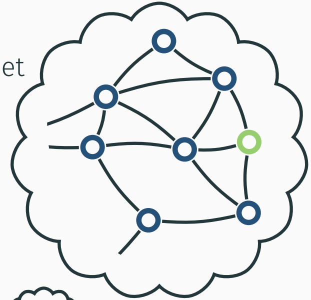
- Every vertex is a dominating vertex or neighbors one
- Choose as few dominating vertices as possible

The LOCAL model

[Linial 1992]

Each vertex decides to be or not in the returned set
Communication only with adjacent vertices





$T = 2$ rounds

Algo $= \mathcal{A}$: distance-T neighborhood $\mapsto$ membership to returned DS

+IDs

Is all hope lost?

Theorem (Kuhn, Moscibroda, and Wattenhofer – 2004)

*It is **impossible** to approximate Minimum Dominating Set with a constant number of rounds and constant approximation ratio on **general graphs**.*

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There are n-vertex graphs which require

$$\Omega\left(\sqrt{\frac{\log n}{\log \log n}}\right)$$

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Restricting the graph class!

State of the art for MDS with constant LOCAL rounds

Graph classes	Approx. Ratio		#rounds
	lower	upper	
trees	3	3	1
outerplanar	5 [2]	5 [2]	2
planar	7 [1]	$11+\varepsilon$ [3]	$f(\varepsilon)$

[1] M. Hilke, C. Lenzen, and J. Suomela, PODC 2014.
[2] M. Bonamy, L. Cook, C. Groenland, and A. Wesolek, DISC 2021.
[3] O. Heydt, S. Kublenz, P. Ossona de Mendez, S. Siebertz, and A. Vigny, European Journal of Combinatorics, 2025.

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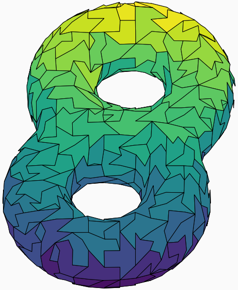

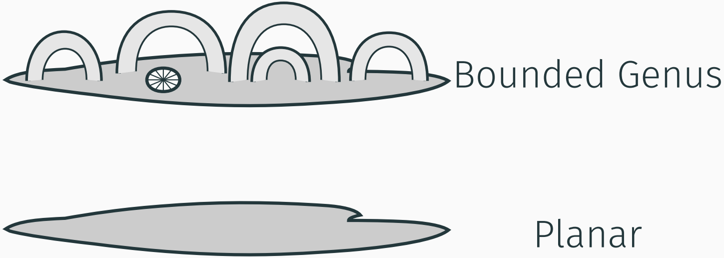

Meta-theorems

Theorem (Bonamy, Das, Gavoille, P, Suomela, Wesolek, PODC'26)

On graphs of Euler genus g, there exists a $(34+\varepsilon)$-approximation of Minimum Dominating Set *in the LOCAL model, in $f(g,\varepsilon)$ rounds.*

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Meta-theorem (baby version)

Any constant-round α-approximation for MDS on planar graphs can be turned into a $f(g)$-round $(3\alpha+1)$-approximation algorithm for MDS on Euler genus-g graphs.

$$\boxed{\mathcal{A}} \xrightarrow{\text{meta-theorem}} \boxed{\mathcal{B}}$$

α-approx
planar graphs
constant-round

$(3\alpha+1)$-approx
genus g
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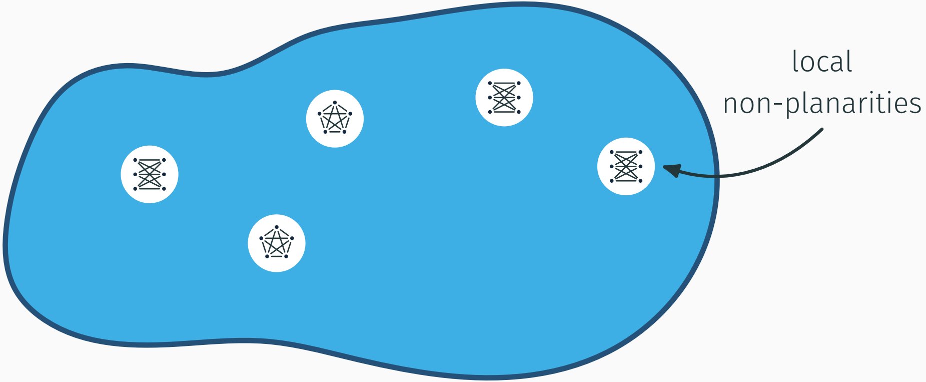
$11 + \varepsilon$ (Heydt et al.) $\quad \boxed{\mathcal{A}} \xrightarrow{\text{meta-theorem}} \boxed{\mathcal{B}} \quad 34 + \varepsilon$

$\mathcal{A}$: α-approx, planar graphs, constant-round

$\mathcal{B}$: $(3\alpha + 1)$-approx, genus g, $f(g)$-rounds

The algorithm $\mathcal{B}$

$T := f(2\rho + 2) + \rho + 2$



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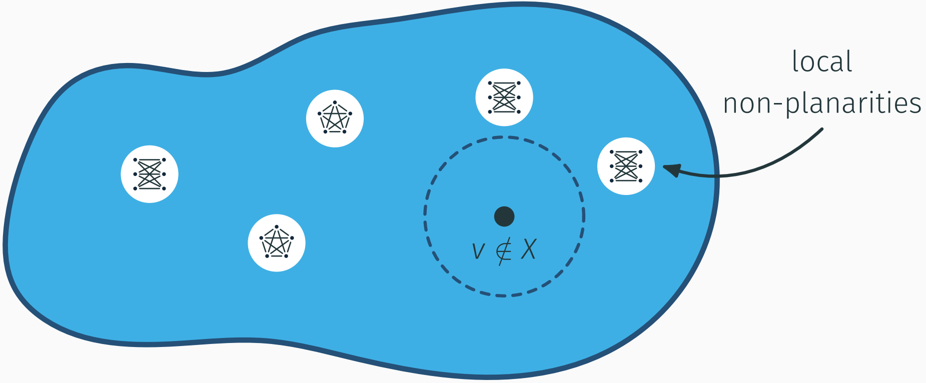

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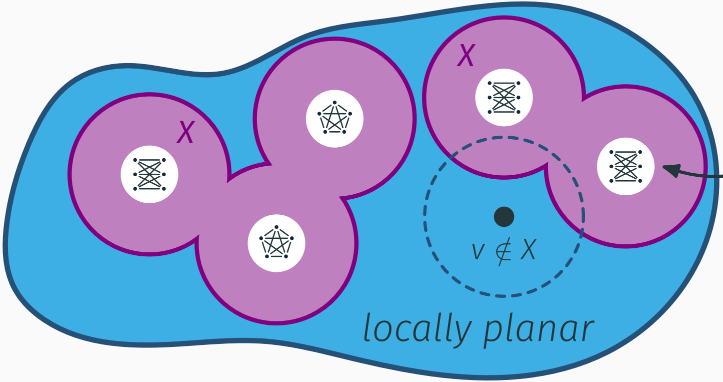

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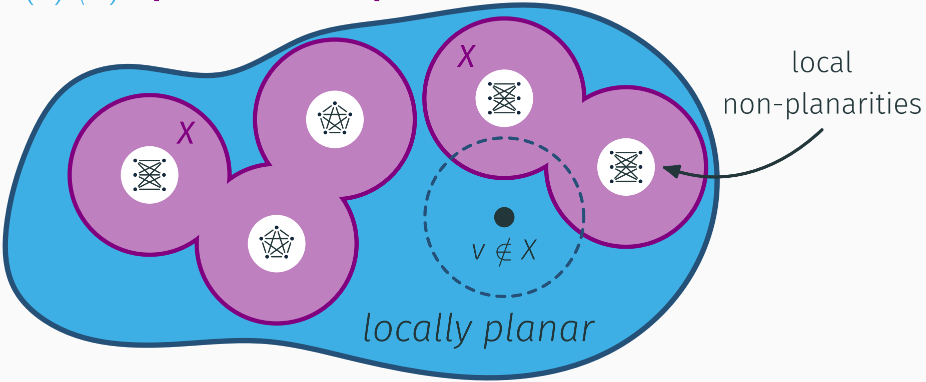

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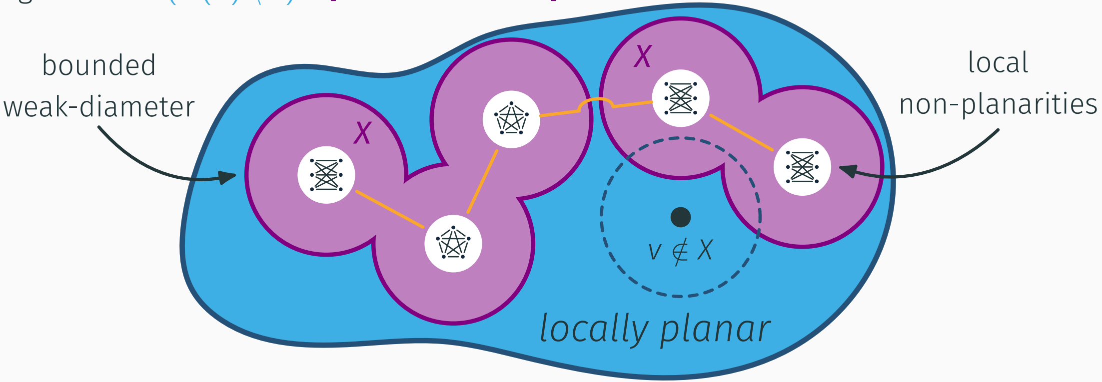

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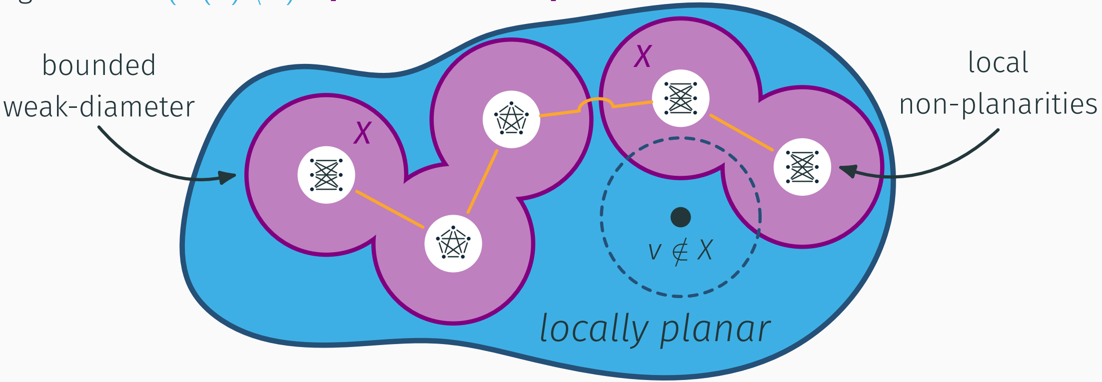
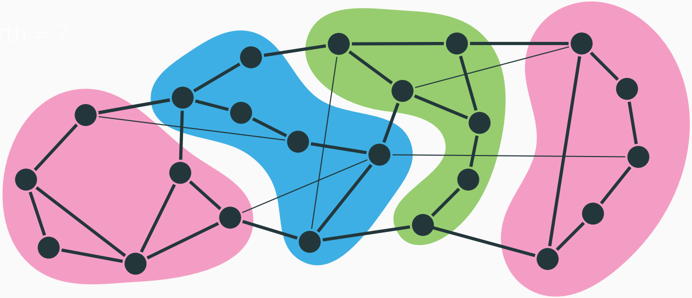
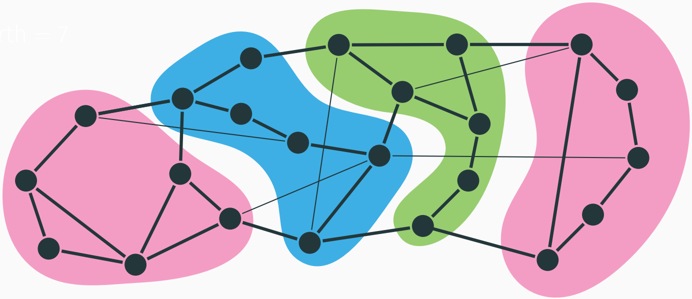


$\text{genus}(\;\cup\;\cup\;\cup \cdots \cup\;) \geq \text{genus}(\;) + \text{genus}(\;) + \cdots + \text{genus}(\;)$



Reusing the algorithm of planar graphs?

Handling locally planar parts



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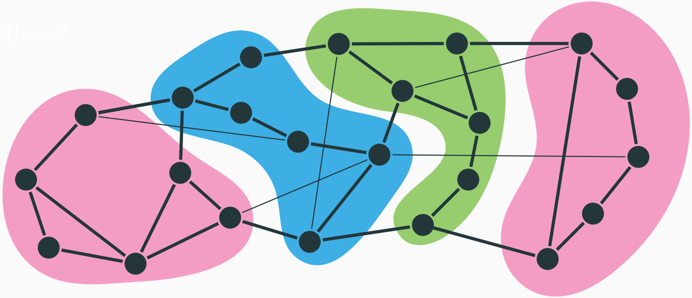

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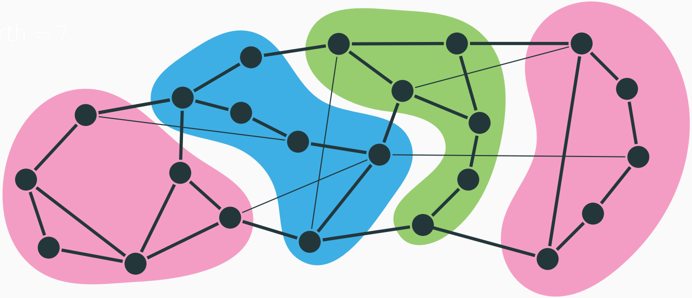

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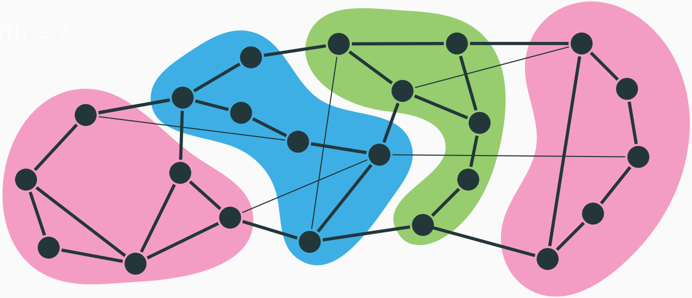

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- Spacing bags of same color ($\implies$ no overcounting for fixed color)
- At most 3 colors ($\implies$ to limit overcounting)

Asymptotic dimension and graph minors

Theorem (Bonamy, Bousquet, Esperet, Groenland, Liu, Pirot, Scott – 2020)

Every class excluding a fixed minor has asymptotic dimension ≤ 2.

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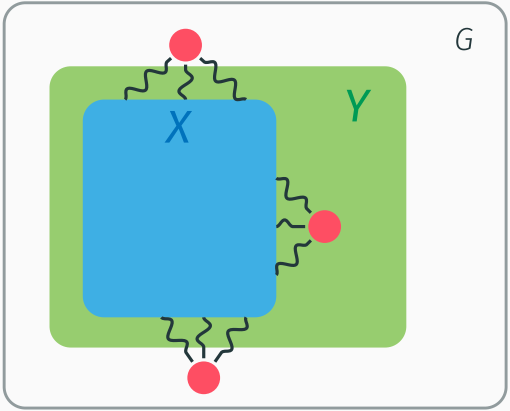
Cuttable problems

$\forall G \in \mathcal{C},\ \forall X \subseteq V(G),\ \exists Y \supseteq X$ such that

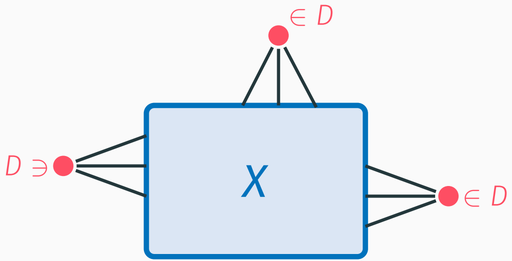
$$\mathrm{OPT}_\Pi(G[Y]) \leq \beta \cdot \mathrm{OPT}_\Pi(G, X)$$

Solving $G[Y]$ is not much harder than solving on X



Proof that MDS is cuttable ($\beta = 1$)

Fix D of minimum size set dominating X in G.



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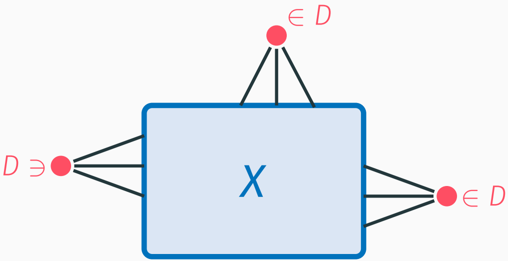

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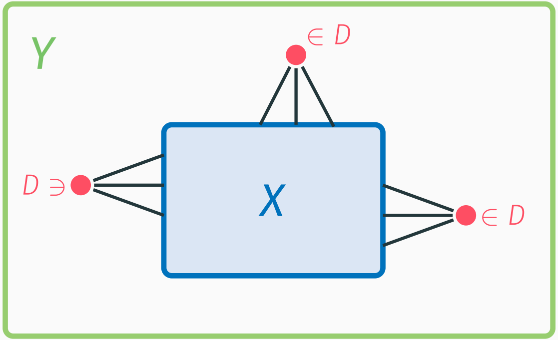

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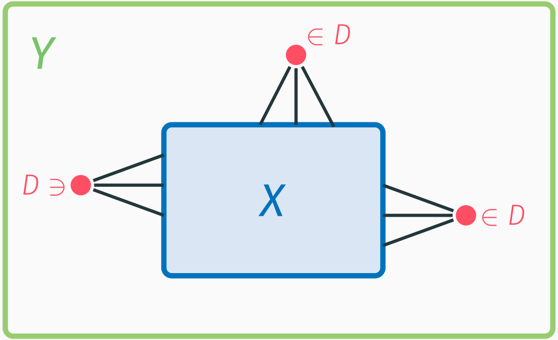

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Let $\boldsymbol{\Pi}$ be a **local** optimization problem that is **cuttable** and **additive**.

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Constant-factor approxs for Minimum k-Tuple Dominating Set, Distance-k Dominating Set, etc...

Conclusion and perspectives

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- Version for when the black-box algorithm requires polynomial IDs.

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